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A FLEXIBLE ANALYSIS PROCEDURE FOR GEOMETRICAL DETECTION OF SPATIAL DEFORMATION

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Abstract

Methods for deformation monitoring are based on two-epoch analysis. During geometrical deformation detection, it is required to transform the results into a common datum, identify a set of stable points and localize the deformation. This paper describes a flexible analysis procedure that incorporates stability determination, congruency testing, S-transformations, localization and testing of spatial deformation. The procedure has been implemented into a computer program, DETECT. Two photogrammetric monitoring schemes were analysed using DETECT for identifying the significance of spatial deformation between epochs. The results confirmed the suitability of the procedure in practical applications.

Introduction

Both the earth's crust and man-made structures (or objects) undergo deformations. Regular monitoring of these deformations is needed to provide information on the stability and extent of any movement or deformation of an object occurring over time. The measuring techniques adopted to measure such deformations are usually divided into geodetic surveys, including ground, photogrammetric and space methods (such as use of the Global Positioning System (GPS)) and geotechnical/structural methods (Chrzanowski and Chrzanowski, 1995).

Frontispiece. BKS Surveys Limited, Coleraine, Northern Ireland has completed a project for Hampshire County Council to provide digital maps of land use and hedgerow categories throughout Hampshire. The land use and hedgerow mapping involved stereoscopic interpretation of contemporary colour air photographs. The land use specification contained over 80 categories which described urban residential, urban non-residential, non-urban residential, quarries, woodlands, agricultural and semi-natural land. Hedgerows, including lines of trees, were classified in six groups based on their height, width, spacing and planting regime.

Georeferenced raster images of the colour photography, with vector overlays of land use and hedgerow categories, have been provided in approximately 200, 25 km^2 digital tiles for the entire county. Staff of Hampshire County Council envisage using the digital maps of land use and hedgerow classes, through a geographic information system, to aid planning decisions across a range of resource management issues.

The illustration shows part of a 1:20 000 scale colour air survey photograph, taken in June 1996 with a Leica RC20 FMC camera ($f = 152$ mm), of an area of the New Forest east of Lyndhurst. Decoy Pond Farm lies in the south-west of the area, Staplewood Hill is in the north-east and Ipley Inclosure is the woodland in the south-east quadrant. The woodland, grassland, heathland and valley mire classifications are shown in part of the accompanying digital tile at a scale of 1:30 000. The three named locations can be identified among the 15 sub-categories of the habitat survey shown in this extract.

BKS Surveys Limited and Hampshire County Council

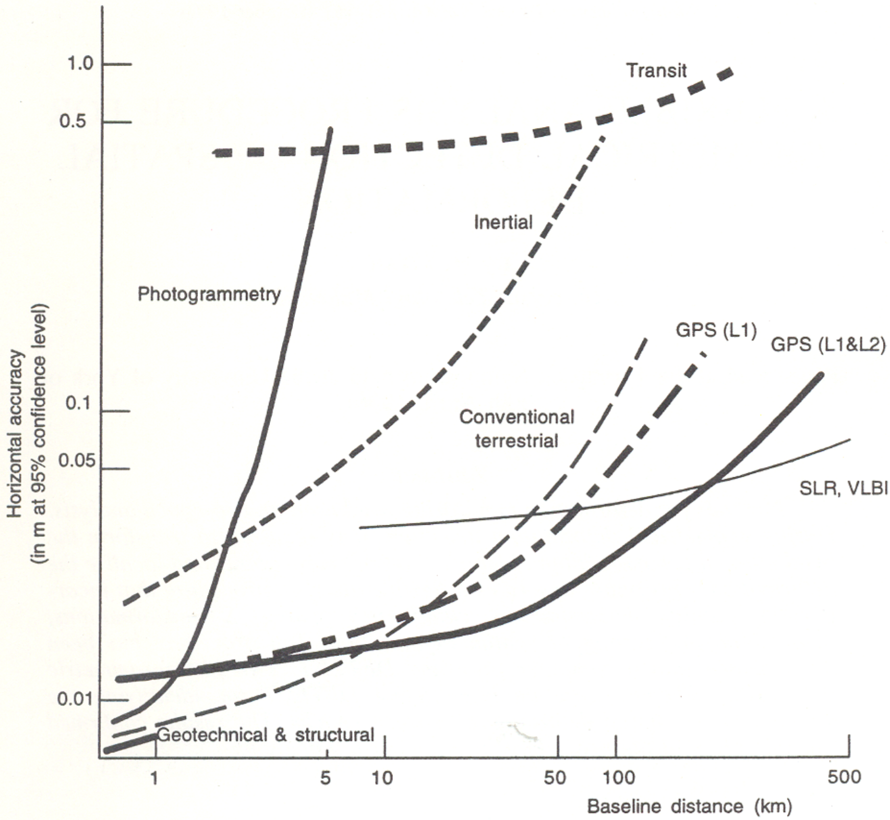

FIG. 1. The performance of measuring techniques (from Wells and associates, 1986).

In general, accuracy decreases with increasing baseline distance (Fig. 1). Geotechnical and structural methods are the most accurate over short distances (Teskey, 1988). However, such methods only provide relative movement within the structures. Geodetic methods are capable of determining overall and absolute movements with respect to a set of stable reference points.

Application of the geodetic method is quite simple. The object under investigation is represented by targeted or marked points. A set of observations is used to connect the points into a monitoring network. The observations are repeated at different epochs of time to provide data for deformation detection. Ground surveys with EDM and theodolite (also total station) are the most commonly used, whilst close range photogrammetry is suitable where multiple targets are involved. The GPS technique is suitable for baselines up to tens of kilometres. Combination of these methods is also possible.

In practice, methods for deformation monitoring are based on two-epoch analysis, consisting of independent least squares estimation (LSE) of a single epoch and geometrical detection of deformation between epochs. Further details on deformation monitoring are given in Niemeier *et al.* (1982), Chen (1983), Gruendig *et al.* (1985), Fraser and Gruendig (1985), Teskey (1986), Chrzanowski and Chen (1986), Caspary (1987), Cooper (1987), Biacs (1989), Teskey and Biacs (1990), Chrzanowski *et al.* (1991) and Setan (1995a).

During geometrical deformation detection, it is required to transform the results into a common datum, identify a set of stable points and localize the deformation. This paper describes a flexible analysis procedure for one-stage geometrical detection of deformation using the geodetic method. The initial sections present the main components of the analysis procedure, followed by their implementation into a computer program called DETECT. Finally, practical applications of the program to analyse two photogrammetric monitoring schemes are highlighted.

Analysis Procedure

The geometrical approach describes the estimated deformation in the form of displacement vectors, without interpretation of the cause of movements. These displacement vectors are referred to a set of common stable datum points between any two epochs. Deformation detection is restricted to common points and datums only. However, in practice, epochs may have different network configurations and datum definitions. It is then necessary to transform the LSE results of each epoch into common stations and datum prior to deformation detection.

During the detection process, no stations are to be assumed stable until tested for stability. Consequently, a method for identifying and testing the stable common points to be used as the datum is needed, followed by the localization of deformations (in other words, the transformation of results with respect to the selected datum points). Statistical testing of the estimated deformations is needed to establish whether significant movements have or have not occurred between the two epochs.

Based on the above needs, the analysis procedure requires the transformation of results into a common datum, identification of a set of stable common points, and localization of deformation together with the appropriate statistical testing.

The analysis procedure developed for geometrical detection of spatial deformation uses one-stage computation, two-epoch analysis, a static model, co-ordinate differencing and an absolute monitoring network. The main modules (Fig. 2) consist of initial checks, preliminary testing on the variance ratio, stability determination by congruency testing, localization of deformation (via decomposition, re-ordering and *S*-transformations) and final testing of deformation using the single point test (Setan, 1995a).

Initial Check

The data required for detection of deformation are obtained directly from the results of the LSE of each epoch, these being the estimated variance factor $\hat{\sigma}_0^2$, degrees of freedom *df*, datum defect *d*, approximate co-ordinates $\boldsymbol{x}_0$, estimated co-ordinates $\boldsymbol{x}$ and their cofactor matrix $\mathbf{Q}_x$.

As an initial check prior to deformation detection, it is important to examine that, for both epochs, the same (common) stations and datum definition (computational base) are being used in LSE. This is very important because of the requirement for common stations and also because $\boldsymbol{x}$ and $\mathbf{Q}_x$ are datum dependent.

If necessary, $\boldsymbol{x}$ and $\mathbf{Q}_x$ can be transformed into the common datum using *S*-transformations prior to analysis as shown in Setan (1995a, 1995c). At this early stage, information related to the common reference points and datum stations needs to be extracted. By rearranging the data of common stations at the beginning of arrays, the extraction process is straightforward.

Computational Stage

Once the initial check is acceptable, preliminary testing on the variance ratio is performed to determine whether the analysis is to be continued. If the test is passed, further analysis consists of one-stage computation for stability determination and localization of deformation.

In these analyses (Setan, 1995a), the initial datum points can be considered as either unknown or known. In the case of unknown datum points, a minimum trace datum is adopted. Otherwise, a partial minimum trace datum is employed, and the datum points can be selected manually. Usually, survey control stations can be used as initial datum stations. In addition, the stability of the initial datum stations can be

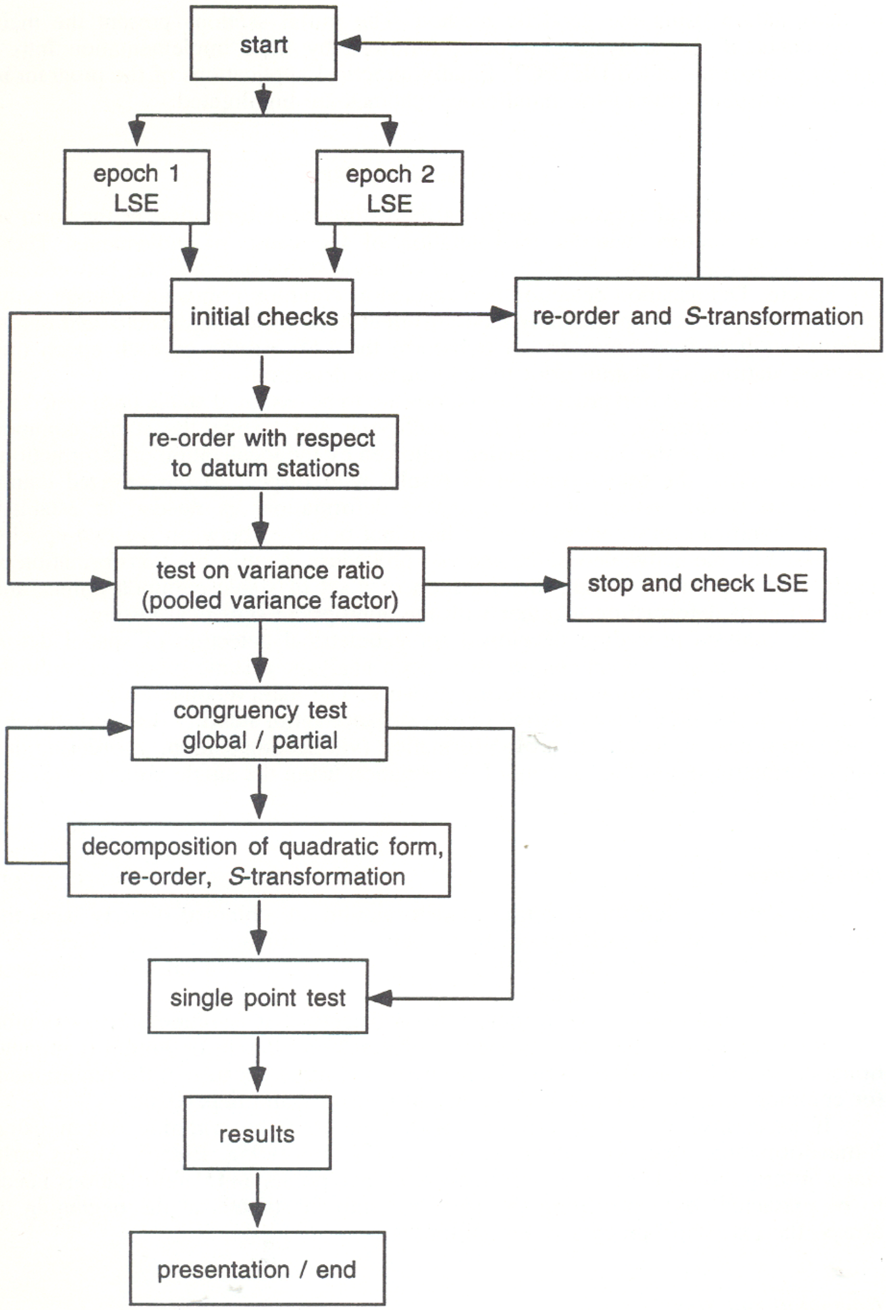

FIG. 2. Procedure for geometrical detection of spatial deformation.

based on some a priori information, such as geotechnical, geological or engineering knowledge.

The procedure for stability determination and localization of deformation employs *S*-transformations (Baarda, 1973; Strang Van Hees, 1982), and is a combination of the Hannover, Karlsruhe and Stuttgart methods. Starting with a chosen computational base (datum), and after appropriate ordering, the datum points are analysed via the methods of congruency testing and localization until the congruency is passed and a set of stable datum points is found. Sequences of congruency testing, decomposition, re-ordering and *S*-transformations are performed iteratively in the analysis. This process is then followed by final testing of deformation via the single point test. During testing, the significance level α can be changed between congruency and single point tests. Standardization of α can be determined computationally or manually.

All the datum stations must pass both the congruency (global) and single point (local) tests (and thus be stable). Otherwise, only the stable datum stations must be used to define the datum, and the detection procedure is repeated.

The above approach is purely geometric and produces vectors of deformations, showing the movement trends. Theoretically, the method requires that three times the number of datum stations be equal to or greater than the number of datum defects, to avoid singularity. In the extreme case of photogrammetric data with a maximum of seven datum defects, a minimum of three stable datum points (hence nine coordinates) are needed.

Test on the Variance Ratio

The test on the variance ratio (Caspary, 1987; Biacs, 1989) examines the compatibility of the independent variance factors of the two epochs. The test can either be one-tailed or two-tailed, with the former, as shown below, being the most commonly used.

$$H_0\colon \hat{\sigma}^2_{0i} = \hat{\sigma}^2_{0j} \text{ at significance level } \alpha \tag{1}$$
$$H_a\colon \hat{\sigma}^2_{0i} > \hat{\sigma}^2_{0j} \text{ or } \hat{\sigma}^2_{0j} > \hat{\sigma}^2_{0i},$$

where $\hat{\sigma}^2_{0i}$ and $\hat{\sigma}^2_{0j}$ are the estimated variance factors of epochs i and j. Let their respective degrees of freedom be df_i and df_j. The test statistic is in the form of a ratio of the variance factors

$$T = \hat{\sigma}^2_{0j}/\hat{\sigma}^2_{0i} \sim F_{df_j,\, df_i} \tag{2}$$

assuming j and i refer to the larger and smaller variance factors respectively. Their relevant degrees of freedom become df_j and df_i. The outcome of the one-tailed test on the variance ratio is

$$\text{if } T < F_{df_j,\, df_i,}\, \alpha, \text{ test passes, accept } H_0 \tag{3}$$
$$\text{if } T \geq F_{df_j,\, df_i,}\, \alpha, \text{ test fails, reject } H_0.$$

If H_0 is accepted, indicating the two variance factors are statistically equivalent, the variance ratio test is passed, and the pooled (or combined or common) variance factor $\hat{\sigma}^2_0$ may be computed as

$$\hat{\sigma}^2_0 = [(\hat{\sigma}^2_{0i})(df_i) + (\hat{\sigma}^2_{0j})(df_j)]/df, \tag{4}$$

where $df = df_i + df_j$.

On the other hand, failure of this test (rejection of H_0) indicates improper weighting of observations, and requires the examination of observational data and/or the LSE results (Chen *et al.*, 1990). The analysis of deformation should be stopped at this stage.

Stability Determination by Congruency Testing

A statistical test known as the congruency test is required to determine whether significant movements have occured between any two epochs. Its purpose is to determine whether or not a set of the "tested" points have moved between any two epochs (Caspary, 1987; Fraser and Gruendig, 1985). The tested points can either be all points common to both epochs (a global congruency test) or selected points used for datum definition (a partial congruency test).

The application of congruency tests is very simple. Initially, the congruency of common datum points at each epoch is tested by the global congruency test. If the test indicates significant movements, localization is performed, followed by a similar test on the remaining (partial) datum points through the partial congruency test. Let the estimated co-ordinates and cofactor matrices for both epochs be $\boldsymbol{x}_1$, $\mathbf{Q}_{x1}$ and $\boldsymbol{x}_2$, $\mathbf{Q}_{x2}$. During deformation detection, it is assumed that these data are referred to a common datum, defined by the same datum points in each epoch.

The outcome of the global congruency test is independent of the a priori selected datum (Caspary, 1987). Hence, either minimum constraints, minimum trace or partial minimum trace datums may be used. However, in general, the minimum trace datum is recommended as the initial datum, if no information on the stability is available. Otherwise, a partial minimum trace datum is used.

At the start of the deformation detection process, the computation of the displacement vector ($\mathbf{d}$) and its cofactor matrix ($\mathbf{Q}_d$) is simply

$$\mathbf{d} = \boldsymbol{x}_2 - \boldsymbol{x}_1 \tag{5}$$

$$\mathbf{Q}_d = \mathbf{Q}_{x2} + \mathbf{Q}_{x1}.$$

The global congruency test (based on Pelzer, 1971) examines the null hypothesis of:

$$H_0\colon \mathbf{d} = \boldsymbol{x}_2 - \boldsymbol{x}_1 = \mathbf{0} \text{ (no significant deformation)} \tag{6}$$

$$H_a\colon \mathbf{d} \neq \mathbf{0}.$$

In other words, the null hypothesis states that the common points in both epochs are stable or have not moved. The test statistic is datum independent (Fraser and Gruendig, 1985; Cooper, 1987).

$$\begin{aligned} T &= \Omega/(h^*\hat{\sigma}_0^2) \\ &= (\mathbf{d}^{\mathrm{T}}\mathbf{Q}_d^+\,\mathbf{d})/(h^*\hat{\sigma}_0^2) \sim F_{h,\,df}, \end{aligned} \tag{7}$$

where

$\Omega = \mathbf{d}^{\mathrm{T}}\mathbf{Q}_d^+\,\mathbf{d}$ is the quadratic form,

$\mathbf{d} = \boldsymbol{x}_2 - \boldsymbol{x}_1$ is the displacement vector,

$\mathbf{Q}_d = \mathbf{Q}_{x2} + \mathbf{Q}_{x1}$ is the cofactor matrix of displacement vector $\mathbf{d}$,

$h = \mathrm{rank}(\mathbf{Q}_d) = 3n - d$ for three dimensional network of n common datum stations and datum defect d,

$\hat{\sigma}_0^2$ = pooled variance factor (4),

$df = df_1 + df_2$ is the sum of degrees of freedom in both epochs (4),

$\mathbf{Q}_d^+$ is the pseudo-inverse of $\mathbf{Q}_d$ such that

$\mathbf{Q}_d^+ = (\mathbf{Q}_d + \mathbf{G}\mathbf{G}^{\mathrm{T}})^{-1} - \mathbf{G}(\mathbf{G}^{\mathrm{T}}\mathbf{G}\mathbf{G}^{\mathrm{T}}\mathbf{G})^{-1}\mathbf{G}^{\mathrm{T}}$ (Caspary, 1987).

Matrix $\mathbf{G}^{\mathrm{T}}$ is the inner constraint matrix. If $\mathbf{G}$ is normalized,

$$\mathbf{Q}_d^+ = (\mathbf{Q}_d + \mathbf{G}_n\mathbf{G}_n^{\mathrm{T}})^{-1} - \mathbf{G}_n\mathbf{G}_n^{\mathrm{T}} \quad \text{as } \mathbf{G}_n^{\mathrm{T}}\mathbf{G}_n = \mathbf{I}.$$

In the deformation detection process, the displacement vectors are usually computed with respect to the first epoch. In computing matrix $\mathbf{G}$, the provisional co-ordinates of the first epoch are used and they are referred to the centroid to avoid numerical instability. Further numerical stability may be obtained by normalizing $\mathbf{G}$ (Setan, 1995a). The above method of computing the pseudo-inverse is adopted because of its simplicity, only involving the inversion of a small matrix. Details are given in Biacs (1989).

The outcome of the congruency test at significance level α is that if T is less than the critical value (that is $T < F_{h,r,\alpha}$), the test passes, and H_0 is accepted. This means that there is no significant deformation within the group of reference points and analysis can be stopped at this stage. Otherwise, if $T \geq F_{h,r,\alpha}$, the test fails and H_0 is rejected, indicating the existence of significant deformation or movement. It is then necessary to examine the nature of the movements via localization, followed by the partial congruency test.

The partial congruency test examines the stability of the partial network formed by the selected or retained datum points only. This is applicable because the set of the retained datum points is actually part of the initial computational base. Let $\mathbf{d}$ and $\mathbf{Q}_d$ be partitioned as (Fraser and Gruendig, 1985)

$$\mathbf{d} = \begin{bmatrix} \mathbf{d}_r \\ \mathbf{d}_e \end{bmatrix}, \quad \mathbf{Q}_d = \begin{bmatrix} \mathbf{Q}_r & \mathbf{Q}_{re} \\ \mathbf{Q}_{er} & \mathbf{Q}_e \end{bmatrix}, \tag{8}$$

where r refers to (retained) datum points, and e refers to non-datum points (datum points eliminated from the computational base).

The null hypothesis for the partial congruency test becomes

$$H_0\colon E\{\mathbf{d}_r\} = 0, \tag{9}$$

or, in other words, the partial network has not changed in shape.

The test statistic

$$T = (\mathbf{d}_r^{\mathrm{T}} \mathbf{Q}_r^{+} \mathbf{d}_r)/((h - 3k)\hat{\sigma}_0^2) \sim F_{h-3k,r}, \tag{10}$$

where k is the number of eliminated points in sub-vector $\mathbf{d}_e$. Interpretation of the test is similar to the global congruency test. If T is less than $F_{h-3k,r,\alpha}$, the test passes, H_0 is accepted and the analysis may be stopped. Otherwise, the test fails, H_0 is rejected and further localization is required. α is the chosen significance level; typically $\alpha = 0{\cdot}05$.

Localization of Deformation

If the global congruency test fails, and hence occurrence of significant deformation is indicated, there is a requirement to locate and isolate suspect unstable points, and at the same time to redefine a new datum for the computations (with respect to datum points). Several methods of localization are available (Chrzanowski and Chen, 1986). The adopted localization procedure has been modified from Fraser and Gruendig (1985).

Starting with a chosen computational base (usually founded on a set of known reference datum points), the procedure removes one point at a time from the computational base. Points are removed via the successive application of decomposition (of quadratic form), re-ordering and partitioning (with respect to the datum points), S-transformations (of $\mathbf{d}$ and $\mathbf{Q}_d$), and partial congruency test until the congruency test passes.

Decomposition of Quadratic Form

If the global congruency test (7) fails (in other words, there is rejection of H_0), the required information on non-congruency between the two epochs is contained in the quadratic form Ω.

In this case, the main task is to investigate the individual contribution of each point (Ω_j) to the total quadratic form Ω. The point with the largest (maximum) Ω_j is usually considered as the most significantly deformed.

The computation of Ω_j is carried out using a decomposition or splitting method. The decomposition procedure is based on Fraser and Gruendig (1985) using techniques of partitioning adopted by Niemeier (1979) and van Mierlo (1981). The vector $\mathbf{d}$ and $\mathbf{Q}_d^{+}$ are partitioned for each point as

$$\mathbf{d} = \begin{bmatrix} \mathbf{d}_r \\ \mathbf{d}_j \end{bmatrix}, \quad \mathbf{Q}_d^{+} = \begin{bmatrix} \mathbf{P}_r & \mathbf{P}_{rj} \\ \mathbf{P}_{jr} & \mathbf{P}_j \end{bmatrix} \tag{11}$$

$$\Omega = \Omega_r + \Omega_j$$

$$\mathbf{d}_j = [dx_j \; dy_j \; dz_j]^{\mathrm{T}}$$

$$\mathbf{d}_{jj} = \mathbf{P}_j^{-1}\mathbf{P}_{rj}\mathbf{d}_r + \mathbf{d}_j$$

$$\Omega_j = \mathbf{d}_{jj}^{\mathrm{T}}\mathbf{P}_j\mathbf{d}_{jj} \text{ for each point } j.$$

The above computation is repeated for each point giving rise to Ω_j. This computational scheme removes the effect of other points in the computed Ω_j, in other words the effect of other points is excluded.

Following this decomposition procedure, the point with the largest Ω_j (considered as the most significantly deformed) is interactively removed from the computational base via re-ordering and S-transformations.

Re-ordering with respect to Datum Points

Re-ordering with respect to datum points is necessary every time a point is removed from the computational base. Re-ordering is also required initially for easy extraction of common and/or reference stations, and for easy manipulation of predefined datum points.

The relevant data to be ordered are displacement $\mathbf{d}$, cofactor matrix $\mathbf{Q}_d$, matrix $\mathbf{G}$ and diagonal matrix $\mathbf{I}_p$. The re-ordering strategy has been established as (12), using the same notation as (8).

$$\mathbf{d} = \begin{bmatrix}\mathbf{d}_r\\ \mathbf{d}_e\end{bmatrix};\ \mathbf{I}_p = \begin{bmatrix}\mathbf{I}_r\\ \mathbf{I}_e\end{bmatrix};\ \mathbf{G} = \begin{bmatrix}\mathbf{G}_r\\ \mathbf{G}_e\end{bmatrix};\ \mathbf{Q}_d = \begin{bmatrix}\mathbf{Q}_r & \mathbf{Q}_{re}\\ \mathbf{Q}_{er} & \mathbf{Q}_e\end{bmatrix}. \tag{12}$$

In the above equation, $\mathbf{d}_e$ refers to the suspected point with significant deformation, that is

$$\mathbf{d}_e = \mathbf{d}_j = [dx_j\ dy_j\ dz_j]^{\mathrm{T}}.$$

$\mathbf{Q}_e$ is a (3×3) symmetric cofactor matrix for $\mathbf{d}_e$. Elements $\mathbf{d}_r$ and $\mathbf{Q}_r$ refer to the partial or remaining datum points. Matrices $\mathbf{I}_p$ and $\mathbf{G}$ are re-ordered to facilitate the use of general equations for the S-transformations (Setan, 1995a) and partial congruency test respectively. Elements of $\mathbf{I}_e$ corresponding to the non-datum points are set to zero, while elements of $\mathbf{I}_r$ (for datum points) remain as one. Matrix $\mathbf{G}$ is computed once only.

S-transformations

The general S-transformations equation has been applied for transforming $\mathbf{d}$ and $\mathbf{Q}_d$ into the new computational base defined by the remaining datum points. The S-transformations scheme of $\mathbf{d}$ and $\mathbf{Q}_d$ into $\mathbf{d}'$ and $\mathbf{Q}'_d$, (12) becomes

$$\mathbf{d} = \begin{bmatrix}\mathbf{d}_r\\ \mathbf{d}_e\end{bmatrix},\quad \mathbf{Q}_d = \begin{bmatrix}\mathbf{Q}_r & \mathbf{Q}_{re}\\ \mathbf{Q}_{er} & \mathbf{Q}_e\end{bmatrix} \tag{13}$$

$$\mathbf{d}' = \begin{bmatrix}\mathbf{d}'_r\\ \mathbf{d}'_e\end{bmatrix} = \mathbf{S}\mathbf{d},\quad \mathbf{Q}'_d = \mathbf{S}\mathbf{Q}_d\mathbf{S}^{\mathrm{T}} = \begin{bmatrix}\mathbf{Q}'_r & \mathbf{Q}'_{re}\\ \mathbf{Q}'_{er} & \mathbf{Q}'_e\end{bmatrix} \tag{14}$$

where $$\mathbf{S} = [\mathbf{I} - \mathbf{G}(\mathbf{G}^{\mathrm{T}}\mathbf{I}_d\mathbf{G})^{-1}\mathbf{G}^{\mathrm{T}}\mathbf{I}_d],\ \mathbf{d}' = \mathbf{S}\mathbf{d};\ \mathbf{Q}' = \mathbf{S}\mathbf{Q}_d\mathbf{S}^{\mathrm{T}}.$$

Elements of $\mathbf{I}_d$ are one (unity) or zero for datum and non-datum points respectively. In the above equations, the elements $\mathbf{G}$, $\mathbf{I}_d$, $\mathbf{d}$ and $\mathbf{Q}_d$ have been ordered. The computational strategy is similar to the evaluation of the S-transformations (Setan, 1995c). Vectors $\mathbf{d}'_r$ and $\mathbf{d}'_e$ refer to the datum and non-datum points respectively.

After this transformation, the remaining network formed by the retained points

($\mathbf{d}_r$) must be tested for stability via the partial congruency test described earlier. The test statistic given by equation (10) can be written as

$$T = (\mathbf{d}_r'^{\mathrm{T}}\mathbf{Q}_r'^{+}\mathbf{d}_r')/((h-3k)\hat{\sigma}_0^2) \sim F_{h-3k,\,df},$$

where k is the number of points removed from the computational base. If the test fails, the process of decomposition of the quadratic form Ω, re-ordering and S-transformations are repeated until the partial congruency test passes.

At the end of the localization stage, $\mathbf{d}_e'$ (14) represents the vectors of deformation of the non-datum points with respect to the datum defined by $\mathbf{d}_r'$. In other words, the solution is in the form of the partial minimum trace datum. The same applies to their cofactor matrix. In order to confirm the localization finding, final testing of deformation may be performed.

As a final confirmation of the localization procedure, the combined LSE of the data from both epochs can be performed, using the stable points ($\mathbf{d}_r'$) as datum. The points in epoch two suspected as significantly moved (in other words, $\mathbf{d}_e'$) are assigned different numbers in each epoch. Alternatively, S-transformations (13) and (14) can be used for this purpose. The vectors of deformation of the non-datum points may be computed directly from their differences in co-ordinates (5) (Setan, 1995a).

Fraser and Gruendig (1985) show that the difference betweeen the vector of deformations from the combined LSE and the final significant deformation vectors obtained by localization ($\mathbf{d}_e'$ in (14)) will be insignificant. Hence, the results of localization can be used directly for demonstrating the deformation trends.

Final Testing of Deformation

Having determined the significant vectors of deformation by means of the localization procedure, the final testing of deformation for verification or confirmation purposes is in the form of a local test known as the single point test. A graphical plot representing the deformation vectors against their point confidence ellipsoid is also useful. The single point test is based on the null hypothesis (Cooper, 1987)

$$H_0\colon \mathbf{d}_j = [dx_j\ dy_j\ dz_j]^{\mathrm{T}} = 0 \tag{15}$$

for each point j.

The test statistic for point j is based on the multi-dimensional F-test

$$T_j = (\mathbf{d}_j^{\mathrm{T}}\mathbf{Q}_{dj}^{-1}\mathbf{d}_j)/(m\hat{\sigma}_0^2) \sim F_{m,\,df}, \tag{16}$$

where m is the dimension of the network. The test statistic for a three dimensional network ($m = 3$) is

$$T_j = \Omega_j/(3\hat{\sigma}_0^2) = (\mathbf{d}_j^{\mathrm{T}}\mathbf{Q}_{dj}^{-1}\mathbf{d}_j)/(3\hat{\sigma}_0^2) \sim F_{3,\,df}, \tag{17}$$

where $\mathbf{Q}_{dj}$ is the cofactor matrix of the displacement vector $\mathbf{d}_j$. This local test is performed for each individual point. If T_j is less than $F_{3,df,\alpha}$, point j is considered as stable, in other words [illegible] displacement vector is not significant. Otherwise it is considered as moved or unstable. In the above test, any correlation between points is

[illegible] expected that all datum points will be stable (the test passes), while [illegible] datum points can either be stable or unstable. Hence, the points with significant movements are expected to be unstable.

The final displacement of each point can be shown graphically, by comparing the displacement vector of each point against its confidence region at a specific significance level. In a three dimensional case, such graphical representation is not straightforward. One approach is by considering the confidence region in all three axes: xy, xz and yz. The displacement vector of any point that lies outside the corresponding confidence region (error ellipses in the xy, xz and yz axes) indicates significant movements. For stable datum points, the plots of displacement vectors will be within the confidence region.

Both the local single point test and plot of deformation vectors (and error ellipses) give similar interpretation. The plot is very useful because it gives an overall picture of any trends in the estimated deformation, both in direction and magnitude. Examination of the plot will also indicate if there are any group movements.

Significance Level in Testing

During statistical testing, the selection of a significance level (α) is to some extent arbitrary. Associated with the test are the Type I and Type II errors. In most applications, only Type I errors are considered. In deformation detection (Biacs, 1989), a Type I error occurs when significant movements are detected but did not occur, in other words a false alarm. Selection of the value of α to be used for global (congruency) and local (single point) tests is quite important. Generally, smaller significance levels are required for the local (single point) test. Standardization of α can be carried out via Bonferroni's inequality.

During the detection of deformation, an arbitrary value of α for the global test may be selected. For a local single point test, neglecting correlation between stations, standardization of α via the application of Bonferroni's inequality (Vaniček and Krakiwsky, 1986) gives

$$\alpha_l = 1 - (1 - \alpha_g)^{1/m} \approx \alpha_g/m, \tag{18}$$

where m is the number of stations, and α_g and α_l are significance levels for global and local tests, respectively. However, as the number of stations is increased, α_l becomes too small, and consequently the critical values become too large. A more practical expression is simply

$$\alpha_l = \alpha_g/(m^{1/2}). \tag{19}$$

In monitoring work, α is often chosen to have values 0·05 and 0·01 for global and local tests respectively. The procedure developed for detection of deformation allows the user to select any value of α for both global and local tests, with automatic standardization via (19) if required.

Program DETECT

Specialized computer programs are required because most commercial software does not allow users to provide the required data for deformation detection purposes. At the Engineering Surveying Research Centre (ESRC) at City University, the following main computer programs (Table I) have been developed for deformation detection (Setan, 1995a).

Table I. The programs and their tasks.

Program	*Task*
GAP	LSE of a single epoch
DETECT	Geometrical detection of spatial deformation
DCRE	Graphical representation of deformation vectors

GAP (developed by Clark, 1992), is capable of processing the combination of photogrammetric and uncorrelated surveying data, and it produces suitable deformation files for DETECT. The program DETECT for deformation detection has been developed by Setan (1995a, 1995b). DCRE is a graphics program developed by Chandler (1994) and uses the plot files created by GAP and DETECT to display the graphics.

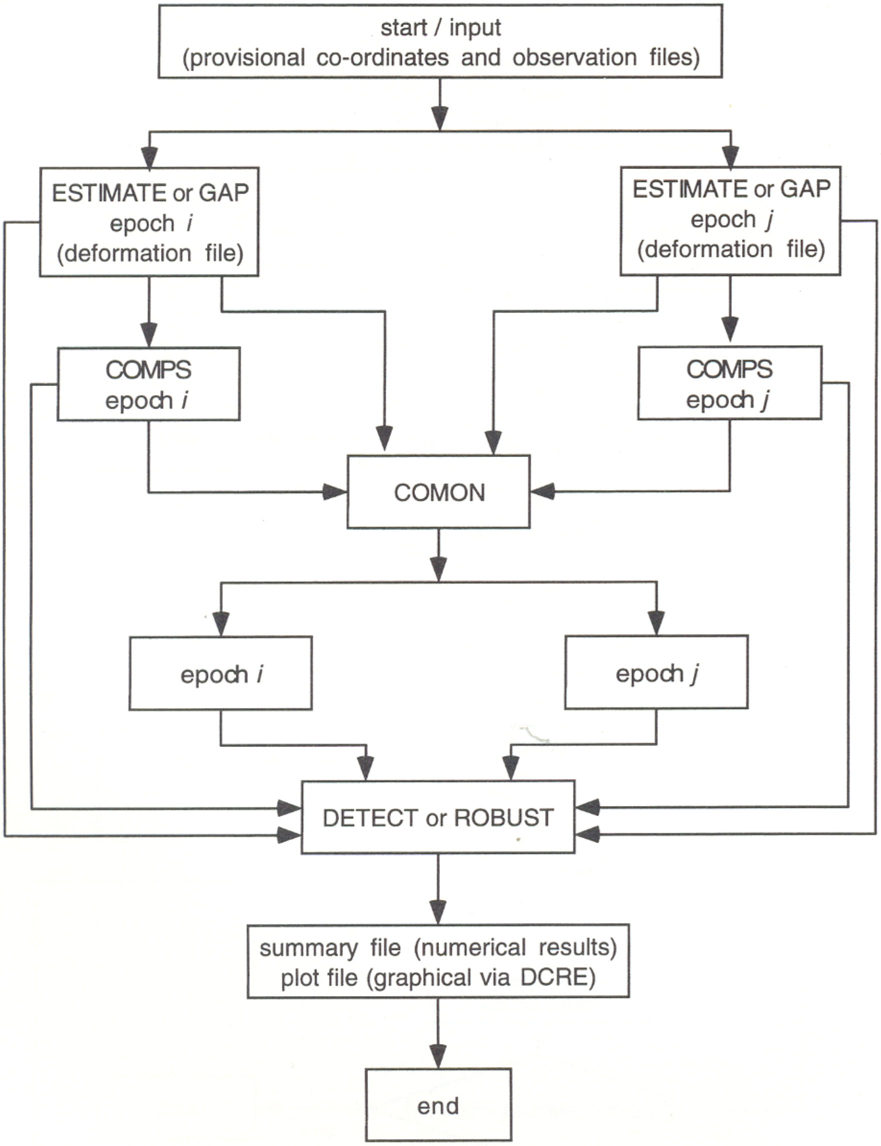

FIG. 3. Linking the programs.

Several other relevant programs are ESTIMATE, COMPS and COMON (Setan, 1995a). ESTIMATE is used for processing surveying observations and it can handle 13 types of uncorrelated observables and three types of algebraically correlated observations. Programs COMPS and COMON were developed for transforming the results of the LSE into the selected datum and common stations respectively, prior to deformation detection. The link between all these programs is shown in Fig. 3.

DETECT can perform a rigorous computation for geometrical detection of spatial deformation, using the relevant LSE results of any two epoch. It employs one-stage computations (1) to (19), excluding (18), for simultaneous analysis of reference and object points. Fig. 4 shows the strategy adopted in DETECT.

Two data input files (deformation files) are required for DETECT, one from each epoch. These files can be obtained from the appropriate output of ESTIMATE, GAP,

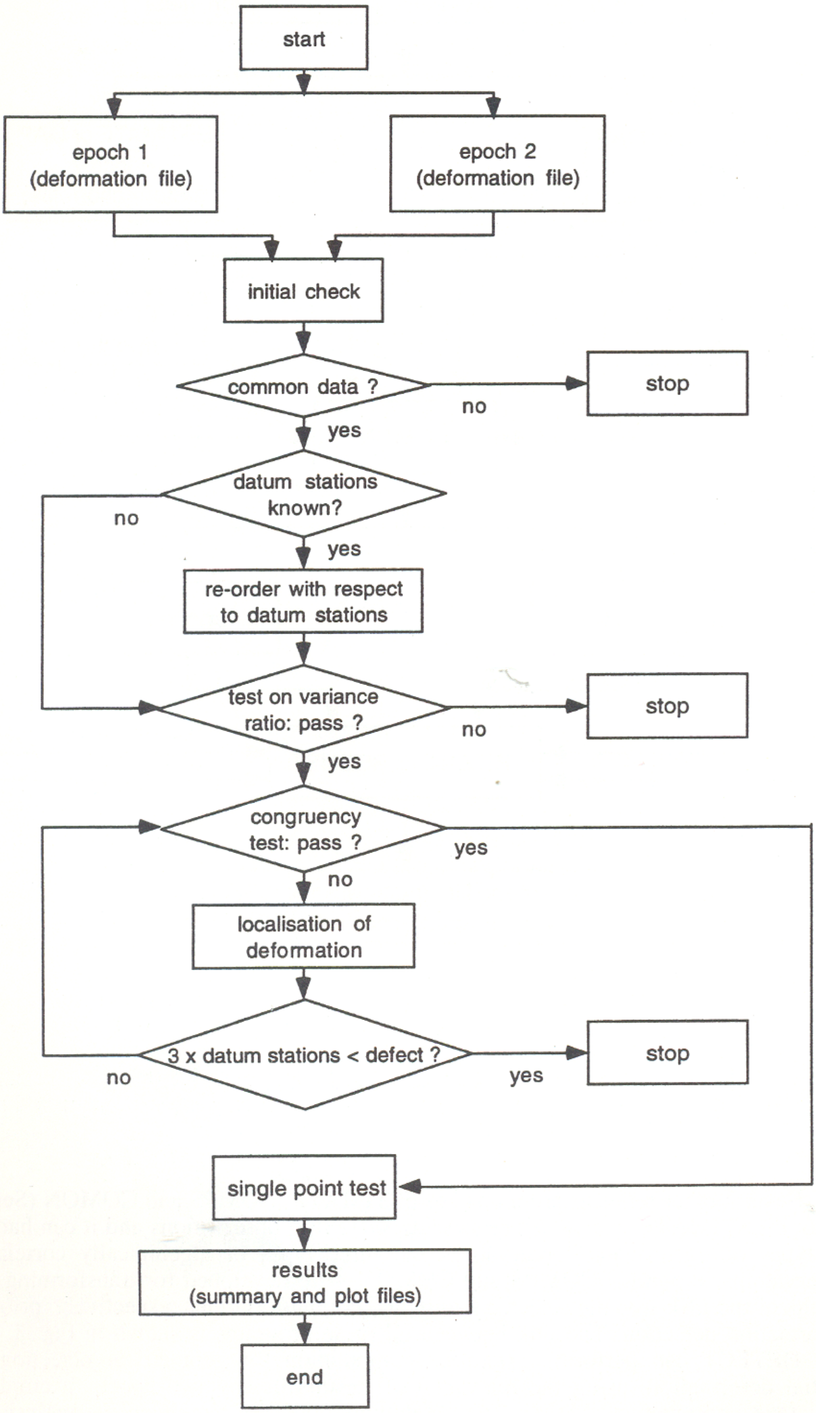

FIG. 4. Flow chart for program DETECT.

COMPS or COMON (Fig. 3). DETECT starts with the reading and checking of input files, selection of datum definition and performance of initial checks on the data. These initial checks ensure that the same common stations, provisional co-ordinates and datum definition are being used in the LSE of each epoch. Any discrepancies will cause termination of the program and, if applicable, are followed by on screen advice for executing COMPS or COMON.

At the start of deformation detection, the datum used for the LSE at each single epoch can be either minimum trace, minimum constraints or partial minimum trace. Datum and non-datum points are related to stable (or reference) and unstable (or object) points respectively. DETECT allows the user to define the status of datum points, whether known or assumed (partial minimum trace) or unknown (minimum trace or minimum constraints) in advance. This can be carried out by editing the input files.

If datum points are not known, all stations will be used for datum definition. Otherwise, the relevant data for the datum stations at each epoch (co-ordinates, cofactor matrix and datum codes) are rearranged at the beginning of their respective arrays for simplicity in further computations (8). A check on the number of co-ordinates used for datum definition against the datum defect is also carried out at this stage, and whenever the datum is redefined. The program is terminated automatically if three times the number of co-ordinates is less than the number of datum defects.

Three statistical tests are employed in DETECT: one-tailed test on the variance ratio, congruency test and single point test. During this testing procedure, the user can select the appropriate significance levels, while approximate critical values are computed automatically. The user also has the option of entering the critical values manually.

Following initial checks, a preliminary test on the variance ratio using (1) to (3) examines the compatibility of the independent variance factors at each epoch. Acceptance of the test leads to computation of common variance factor (4) and stability determination. Failure of the test will terminate the program automatically, and requires the examination of the LSE results and observational data.

Stability determination starts with checks on the stability of initial datum points in both epochs via congruency tests (5) to (7). If the test indicates significant movements of the datum points, localization of deformation is performed. Stability determination and localization of deformation consist of an iterative process of congruency testing (9) and (10), decomposition of the quadratic form (11), re-ordering with respect to datum points (12) and *S*-transformations (13) and (14) until the partial congruency test passes. During this iterative procedure, the datum point with the largest quadratic contribution is removed from the computational base.

Once the congruency test passes, the localization procedure will compute any significant deformation vectors of non-datum points with respect to the final datum points. Final testing of deformation is in the form of a single point test (15) to (17). DETECT computes the standardized significance level for this test via equation (19), and also allows the user to change the significance level and its critical value.

In DETECT, approximate co-ordinates are used for computing matrix **G**. Computation of displacement is with respect to the first epoch. The program also provides automatic and manual modes of the above iterative procedure. In the manual mode, the user can also select which datum point is to be removed from the computational base. This facility is not possible in automatic mode. During datum re-definition, a check against datum defect is also carried out. Another useful feature of DETECT is the direct comparison between two epochs. In this mode, each epoch must be based on the same datum (Setan, 1995a).

DETECT will produce summary and plot files, as selected by the user. The summary file (compulsory) contains the full results of deformation detection, whilst the plot file contains useful information for DCRE. In the plot file, data for each station consist of station names, co-ordinates, deformation vectors, and their respective sub-cofactor matrix. By using DCRE, the deformation vectors and error ellipses (in three axes) can be portrayed graphically.

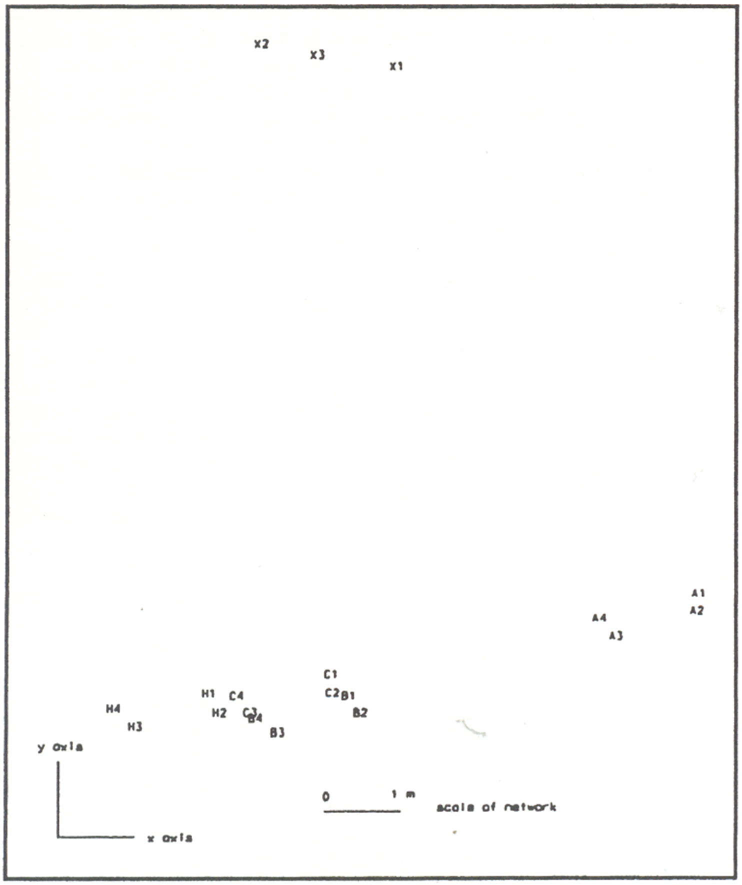

FIG. 5. 19 station network (plan view).

RESULTS

Two photogrammetric monitoring networks have been used to determine the suitability of DETECT for deformation detection. The significance level for testing was chosen as 0·05, except for the single point test, where a significance level of 0·01 was used.

Photogrammetric Monitoring Network

A plan view of a 19 station photogrammetric monitoring network, with seven datum defects, is shown in Fig. 5. Deformations were simulated at stations $A1$ as $(-0{\cdot}100, 0{\cdot}050, 0{\cdot}030)$ m and $B1$ as $(-0{\cdot}100, 0{\cdot}100, 0{\cdot}100)$ m to generate data for a second epoch.

In each epoch, a LSE with minimum trace solution was obtained using GAP. Each solution was found to pass the global test. The estimated variance factors and degrees of freedom for the first epoch were 1·1963 and 128, whilst those for the second epoch were 1·1779 and 94. Special files for deformation detection were created by GAP.

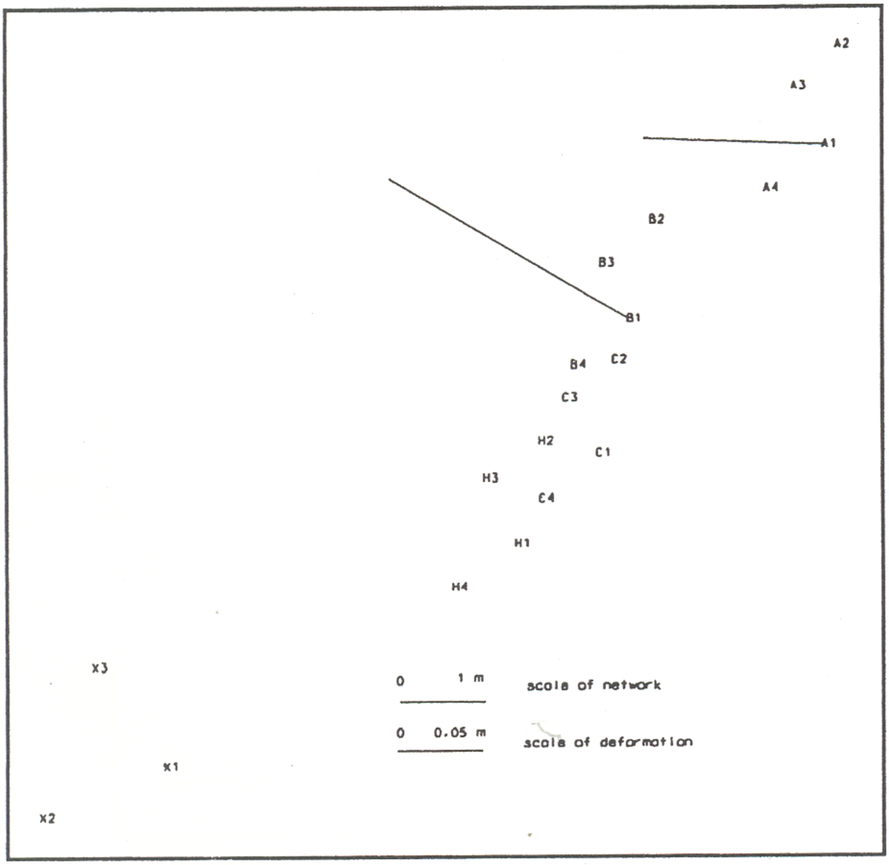

FIG. 6. Estimated deformation for 19 station network (isometric view).

Results of deformation detection from DETECT are very close to the simulated deformation. Starting with 19 datum stations, DETECT resulted in 15 stable datum stations and 4 non-datum (1 stable and 3 unstable) stations. The unstable stations were found to be *A*1, *B*1 and *A*2. Station *C*1 is stable, although being actually a non-datum point. All the datum stations were confirmed as stable, passing the single point test. Station *A*2 is flagged as unstable because its computed statistic for the single point test was bigger than the critical value at $\alpha = 0{\cdot}01$. *A*2 has a small deformation vector (0·003 m) compared to stations *A*1 and *B*1 and, although significant, is small. Significant deformations for stations *A*1 and *B*1 are shown in Table II.

Differences between the simulated and estimated deformations are very small, indeed almost negligible. Graphically (output of program DCRE), Fig. 6 shows the

TABLE II. Significant deformations for two stations.

Data source	*Deformation: station A1 (m)*	*Deformation: station B1 (m)*
Simulated	(− 0·100, + 0·050, + 0·030)	(− 0·100, + 0·100, + 0·100)
From DETECT	(− 0·101, + 0·048, + 0·031)	(− 0·099, + 0·100, + 0·101)

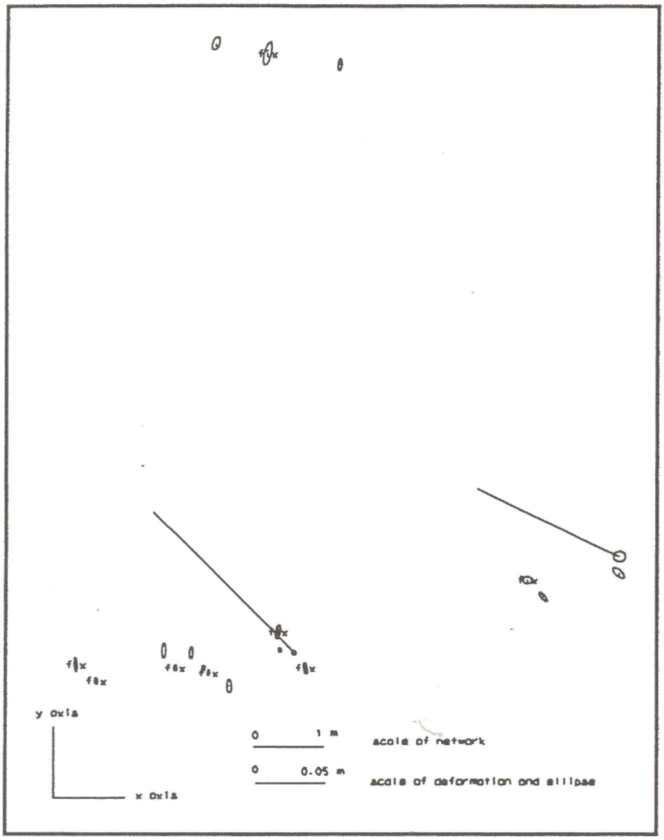

FIG. 7. Estimated deformation of 19 station network (plan view).

displacement of stations $A1$ and $B1$. The results for all three sets of axes are shown in Figs. 7 to 9, where the deformation vectors of stations $A1$, $B1$ and $A2$ are outside the ellipse.

Wood Panel Study

A deformation study has been carried out on a wood panel in order to investigate the effect of temperature and humidity on the behaviour of wood panels. Such panels are used widely as frames for the preservation of paintings. The study is being carried out by ESRC at Hamilton Kerr Institute, Cambridge.

This investigation is still active; it involves several epochs and various types of wood panels. A special room that allows temperature and relative humidity to be controlled is used. In general, each panel, with pre-marked points, is slotted into a fixed frame and then imaged by five digital cameras. Data are automatically transferred into a PC, using the concept of a three dimensional measuring system. Automated methods of target location, identification and matching are followed by the LSE of camera parameters and object co-ordinates. Details of the data collection procedure and LSE are given in Robson *et al.* (1995).

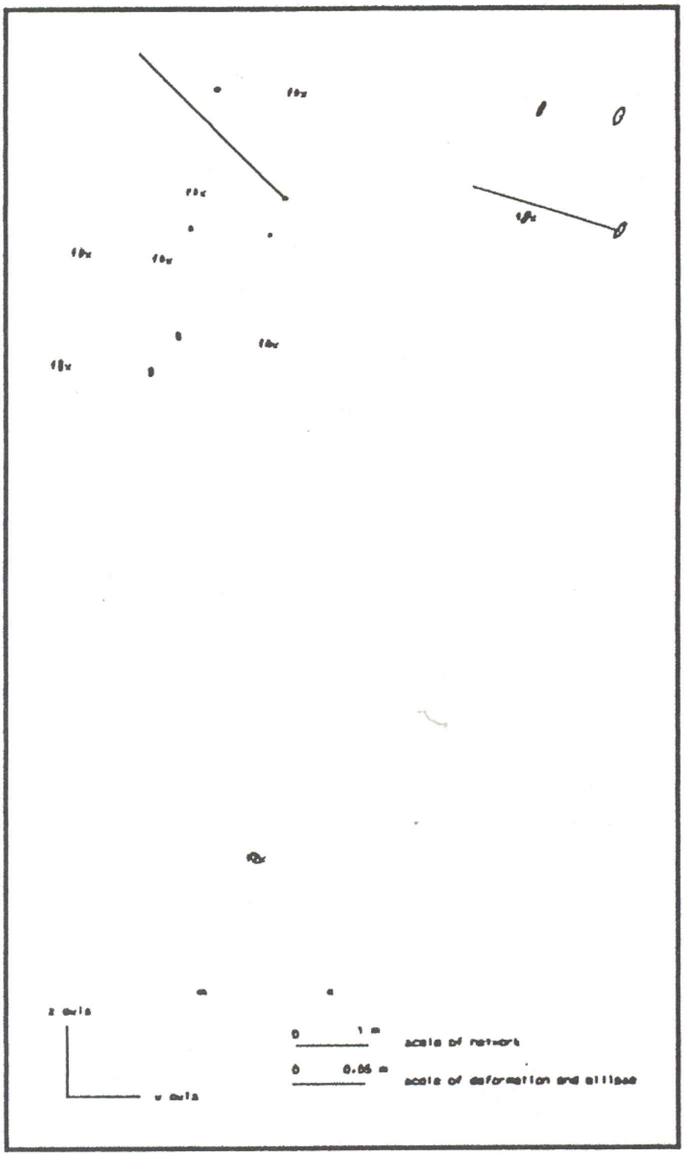

FIG. 8. Estimated deformation for 19 station network (front view).

Data from two epochs, that comprised 169 points in each epoch, were analysed (Fig. 10). There are 12 points situated on the fixed frame (*A*1, *A*2, *A*3, *B*1, *B*2, *B*3, *C*1, *C*2, *C*3, *D*1, *D*2 and *D*3), whilst the remaining 157 points are on the panel. In the second epoch, the room temperature and relative humidity were increased drastically. During the LSE, the number of degrees of freedom were 1154 and 1156, whilst the estimated variance factors were 0·6835 and 0·5497, for the first and second epochs respectively.

Initially, the 12 points on the fixed frame were used to define the starting datum for deformation detection. The results of the detection indicated station *B*2 as

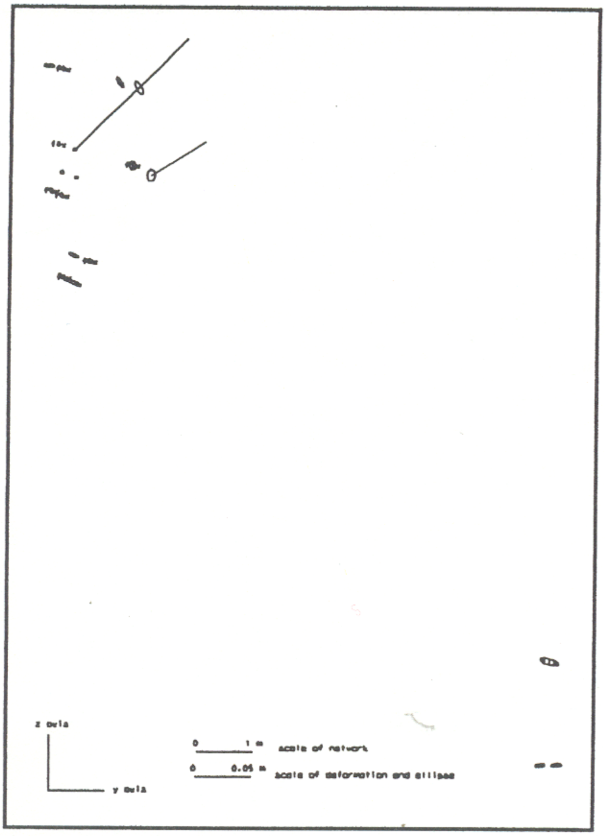

FIG. 9. Estimated deformation for 19 station network (right view).

unstable, 11 stable datum stations and 158 non-datum stations (16 stable, 142 unstable). The pattern of the deformation is displayed in Figs. 11 to 13, indicating the significant upward movement of both the left and right sides of the panel. The maximum displacement vector (18·1 mm) and movement in the z direction (18·1 mm) were detected at station 233. Maximum movements in the x and y directions were found at points 258 ($-0{\cdot}7$ mm) and $B2$ ($-0{\cdot}8$ mm) respectively.

CONCLUSIONS

An analysis procedure for geometrical detection of spatial deformation using a geodetic method has been demonstrated. For flexibility, the main features of the procedure include one-stage computations, manual selection of datum stations, automatic computation mode, change of significance level between global and local tests, and direct comparisons between epochs.

The procedure has been implemented into the computer program, DETECT. The results obtained show that the developed strategy and programs are applicable for the geometrical detection of spatial deformation.

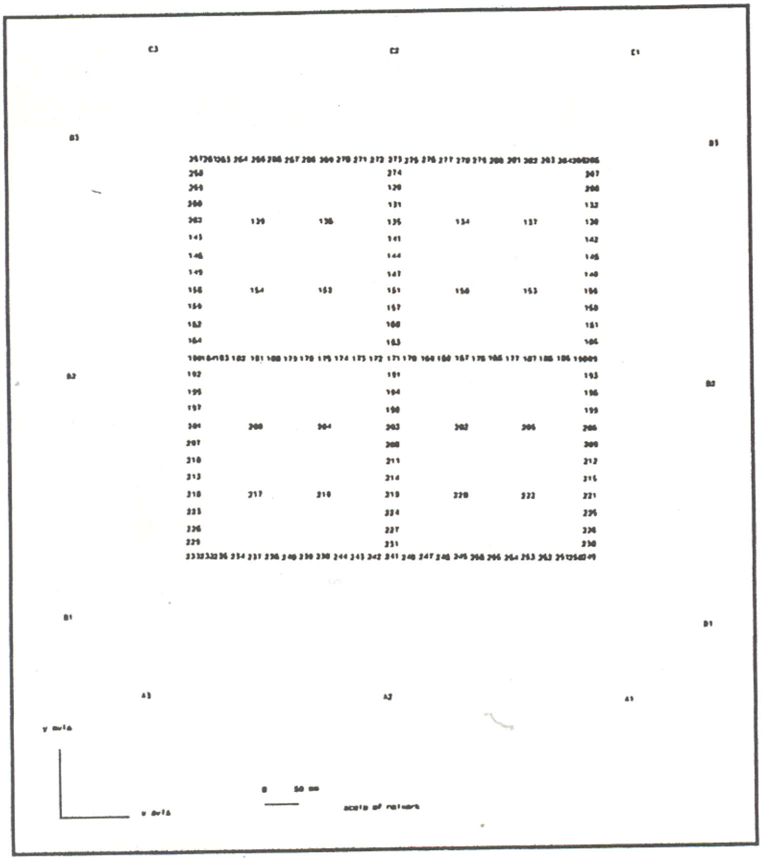

FIG. 10. Wood painting network (plan view).

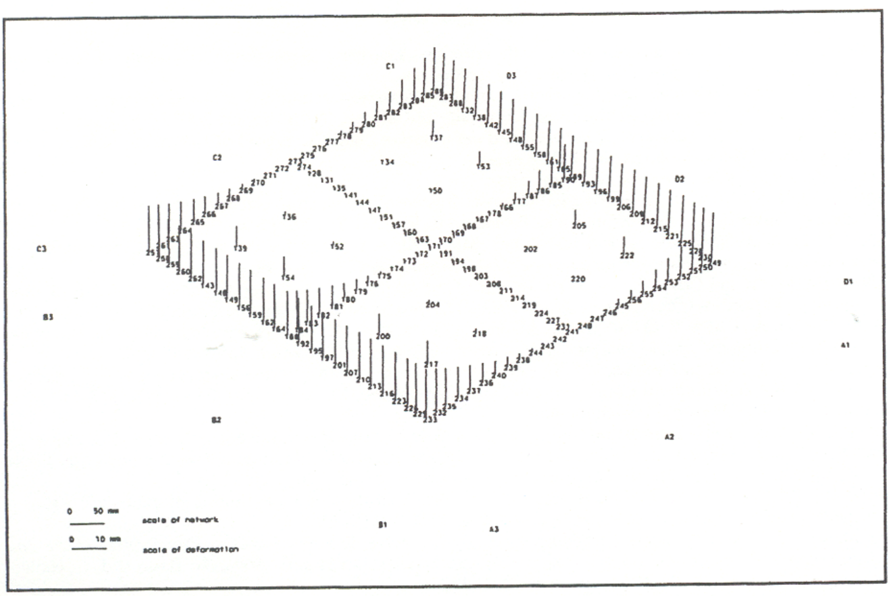

FIG. 11. Estimated deformation for wood painting network (isometric view).

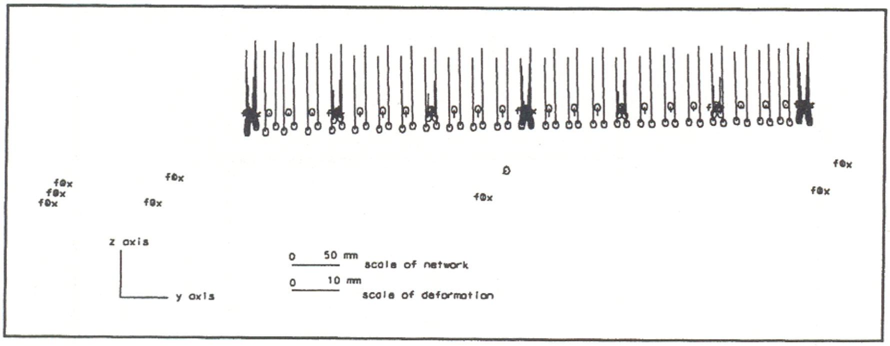

FIG. 12. Estimated deformation for wood painting network (right view).

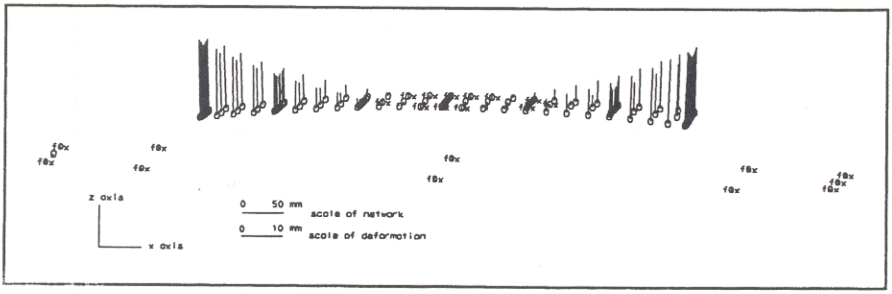

FIG. 13. Estimated deformation for wood painting network (front view).

To date, the procedure has been successfully applied in many real monitoring applications using standard and digital close range photogrammetric systems (Setan, 1995a; Robson *et al.*, 1995; Robson and Cooper, 1995; Robson, 1996).

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Résumé

Les méthodes utilisées pour suivre les déformations sont basées sur des analyses à deux époques. Pour détecter les déformations géométriques, il est nécessaire de transformer les résultats dans un référentiel commun, d'identifier un jeu de points fixes et de localiser ces déformations.

On présente dans cet article une méthode d'analyse souple qui comprend la réalisation de la stabilité, la recherche des congruences, les transformations de coordonnées, la localisation et l'analyse de la déformation dans l'espace.

Cette méthode a fait l'objet du logiciel d'ordinateur DETECT. On a étudié deux schémas de suivi photogrammétrique utilisant DETECT pour mettre en évidence de façon significative la déformation dans l'espace entre deux époques.

Les résultats ont confirmé que la méthode convenait tout à fait en pratique.

Zusammenfassung

Die Verfahren des Deformationsmonitoring basieren auf der Zweiepochenanalyse. Bei der Aufdeckung geometrischer Deformationen ist es erforderlich, die Ergebnisse auf einen gemeinsamen Bezug zu transformieren, einen Satz stabiler Punkte zu identifizieren und die Deformation zu lokalisieren. In dem vorliegenden Beitrag wird ein flexibles Analysenverfahren beschrieben, das die Stabilitätsbestimmung, die Kongruenztestung, S-Transformationen sowie die Lokalisierung und Testung räumlicher Deformationen umfaßt. Das Verfahren wurde in ein Rechnerprogramm namens DETECT implementiert. Zwei photogrammetrische Monitoring-Schemata wurden unter Nutzung von DETECT zur Identifizierung der Signifikanz räumlicher Deformationen zwischen Epochen analysiert. Die Ergebnisse bestätigen die Eignung des Verfahrens bei praktischen Anwendungen.